Is MRW model still applicable for oil-based economies? Evidence from Libya

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Abstract:

This paper investigates weather MRW model can be applied for oil-based economies and if it is applicable for time-series data. MRW model was suggested by Mankiw, Romer, and Weil based on the basic formula originated by Robert Solow 1956. MRW argued that human capital plays a major role in modern economies depends on developments that occurred in the developing countries. They arrived at human capital contributing much more to the economic growth instead of neoclassic idea. MRW model had been applied empirically for developing and under-developing countries however, the oil-based economies were excluded and never applied in this context. This study attempts to fill this gap whether this formula offers an appropriate tool to explain the growth in Libyan economy as a case of natural-sourced economies. With data spanning over five decades the GDP is estimated applying both models and FMOL to avoid time-series problems, we found that MRW model is unable to explain the changes in per-capita output while the original model of Solow did. Moreover, the role of human capital is unclear. Result supports the initial suggestion of Mankiw, Romer and Weil. Meanwhile, as the basic formula with only physical capital does explain the changes while the MRW model does not.

Key words: Growth, Country Studies, Aggregate Model, Time-series Models, Resources and Growth.

هل يمكن تطبيق نموذج MRW للنمو الاقتصادي على الاقتصاديات النفطية؟ حالة الاقتصاد الليبي

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الملخص:

لايزال موضوع النمو الاقتصادي يثير الكثير من الجدل في الأدب الاقتصادي. ويعد النموذج الذي اقترحه (سولو Solow) لتفسير تغيرات الناتج الكلي أحد النماذج البارزة في أدبيات النمو الاقتصادي ويحتوي فقط متغيرين هما عنصري العمل ورأس المال وهو ما يتوافق مع دالة الإنتاج التقليدية ثم أضاف شكلاً موسعًا يحتوي متغيّرًا ثالثًا هو (رأس المال البشري H) كمتغير مفسر، وقد قام كلا من (مانكيو، رومر، وايل Mankiw, Romer and Weil) بتجربة النموذج قياسيًّا وتوصلوا إلى نتائج أفضل لتفسير تغيرات الناتج الكلي في عدد كبير من الدول. لكنهم استبعدوا من دراستهم الاقتصاديات المعتمدة على النفط وعرف هذا النموذج باسم MRW.

هذه الورقة تحاول استكشاف ما إذا كان النموذج الموسع لصولو قابلاً للتطبيق في اقتصاد ريعي يعتمد على النفط مثل ليبيا خلافًا لما افترضه MRW؟ كما تستهدف الورقة أيضًا اختبار قابلية النموذج نفسه للتطبيق على بيانات السلاسل الزمنية إذ لم يتم مثل هذا الاختبار سابقًا.

ولذلك فقد تم تقدير الناتج القومي في ليبيا لفترة تمتد 58 عامًا منذ 1962 وحتى عام 2020م وهي الفترة التي تغطي الوفرة النفطية وذلك باستخدام النموذجين المذكورين لصولو وبتطبيق منهجية FMOLS للتقدير لمعالجة مشاكل الاشتراك الخطي والارتباط الذاتي المتوقعة في بيانات السلاسل الزمنية. وتبيّن من النتائج أن فرضية صعوبة تطبيق نموذج صولو الموسع على اقتصاد ريعي كما افترض مانكيو ورومر ووايل صحيحة إذ لم يتمكن النموذج الموسع من تفسير تغيرات الناتج للفترة المذكورة بينما نجح النموذج الأول في ذلك.

الكلمات المفتاحية: النمو الاقتصادي، دراسات قطرية، نماذج الاقتصاد الكلي، نماذج السلاسل الزمنية، الموارد الناضبة والنمو.

1. Introduction.

The main aim of this paper is to investigate whether MRW model can be applied to explain changes in output for Libyan economy as an oil-rich one? This topic has received little attention in the empirical literature in the Libyan economy.

Since Robert Solow (1956) suggested his model for endogenous growth, plenty of studies have enriched this field. Mankiw et al., (1992) contributed the most important one by emphasizing Solow's main idea about exogenous growth. They developed a new formula—known as the Mankiw, Romer, and Weil (MRW) model—by adding an independent factor (Human capital [H]) to the formula, to be three independent variables. They examined it for 98 countries, both developed and developing, and claimed that it is consistent with variations in the standard of living across t1he world. They also argued that the new formula provides an excellent description of cross-country data, assuming that elasticities of the term Y/L with respect to savings [s] and $(\eta + \delta + \lambda)$ (population growth plus capital depreciation plus technical change) will be around (0.5) and (–0.5), respectively. It predicts not only the signs but also the values of coefficients on savings and population growth. Therefore, this assumption provides a good tool to judge both models (i.e., Solow and MRW). If the estimated parameters are different substantially from these values, then Mankiw, Romer, and Weil are allowed to reject the joint hypothesis that the Solow model and their identifying assumption are correct. It is worth to mention that Mankiw, Romer, and Weil have empirically excluded the oil-producing countries because they think that one should not expect a standard model of growth to account for bulk gross domestic product (GDP) (Mankiw et al., 1992). Moreover, they applied only cross-sectional data for their study. Therefore, it is worth to clarify whether the MRW formula applies to oil-based economies, and whether it is applicable to any specific country (i.e., time-series data). Thus, the main objective of this paper is to answer the following questions. To what extend the MRW formula is able to explain the changes in per-capita income for a specific country? Is MRW model still applicable even for non-renewable resources? what are the shares of labour and capital in Libya?

To the best of my knowledge, this model has been not examined for time-series data nor for the oil-based economies. Since Libya offers a good example for our goal as an oil-based economy. It is important to test this model in this context. The remainder of the study consists of four sections: first, a brief description of the Libyan economy; second: methodology, data and estimation, third, a discussion of obtained results. Finally, concluding remarks.

2. The Structure of the Libyan Economy and Input Factors.

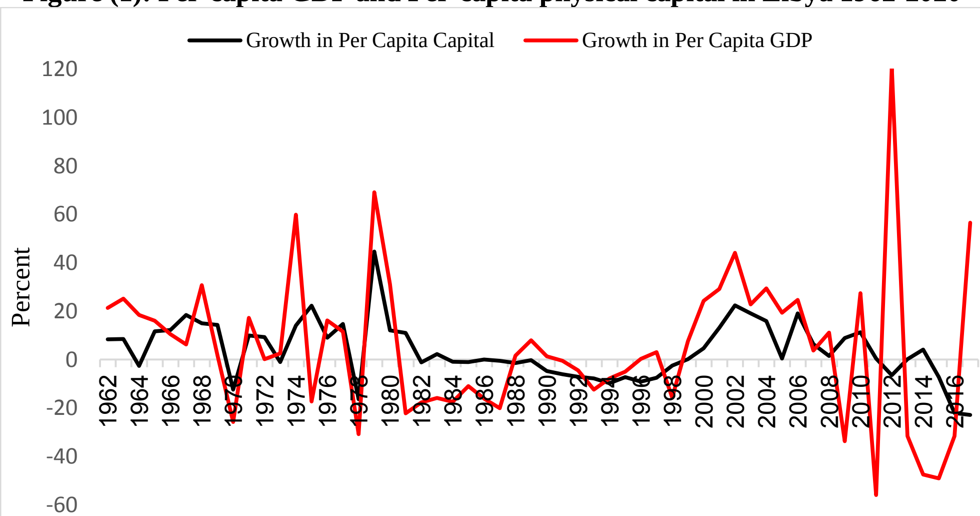
In the 1960s, funds became available for investment in Libya. This was chiefly related to oil extraction rather than to other economic activities. Ruhaet (2013) confirmed that the growth was barely sustained during the subsequent global oil shocks and the industry experienced negative growth in many years.

Ahmouda (2014) asserted that although growth in the non-oil sector was greater than that in oil, most of the economic growth was attributed to oil revenues. Physical capital and labour force grew at 6.9% and 4.24%, respectively, on average over the period; both grew greater than aggregate and non-oil GDP which both had grown by 4.2% and 0.14%, respectively.

A. Physical Capital:

Although the average real growth of physical capital over the period 1962-2020 was 6.71%, it can be noticed that the growth in physical capital followed the growth pattern in GDP throughout the period as demonstrated in the graph (1). Both physical capital per worker and GDP per worker showed a similar trend, and both were affected by oil shocks. Capital per worker grew following the oil revenues in most years.

Figure (1): Per-capita GDP and Per-capita physical capital in Libya 1962-2020



Source: Author's work.

More specifically, per worker physical capital, which considered a crucial indicator to promote economic growth showed a similar trend of per-worker output (Romer, 2012). Both were subjected to a remarkable decline due to a dramatic fall in oil revenues during the eighties. Also, due to unstable investments and the increase in governmental workers because of intervention policies adopted in that time.

A decline in real physical capital per worker between 1983 and 2000 was due to investments being mostly dedicated to social needs; therefore, the investment–output ratio witnessed a long run downward. As in Figure 2, This makes the role of capital

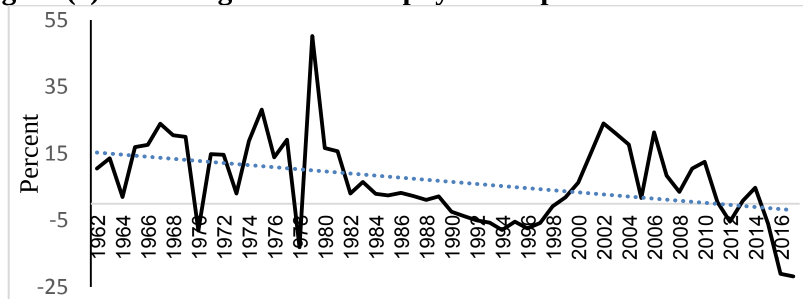
Figure (2): Physical capital per worker K⁄L (Real L.D)

Source: Author's work. Values are in real Libyan Dinars.

Capital per worker grew at a higher rate than per-capita GDP in some years. However, this trend should be questioned in terms of the investment method, which focused on necessary social infrastructure[1] rather than building a productive economic structure and widening the production ability of the whole society. This may explain the decline in real physical capital per worker between 1983 and 2000 when available funds became short and confirms the priority of basic social needs in Libyan society over the production structure and recent social requirements are considered at the expense of investing for the future.

1- Between 1973 and 1985 the government adapted a plan covered the whole country to fund roads, housing, high voltage electricity net and alike, however these investments theoretically are part of physical capital, many of them are out of usage or unreliable due to the way they located in a such large country.

Figure (3): Annual growth in real physical capital stock 1962-2020



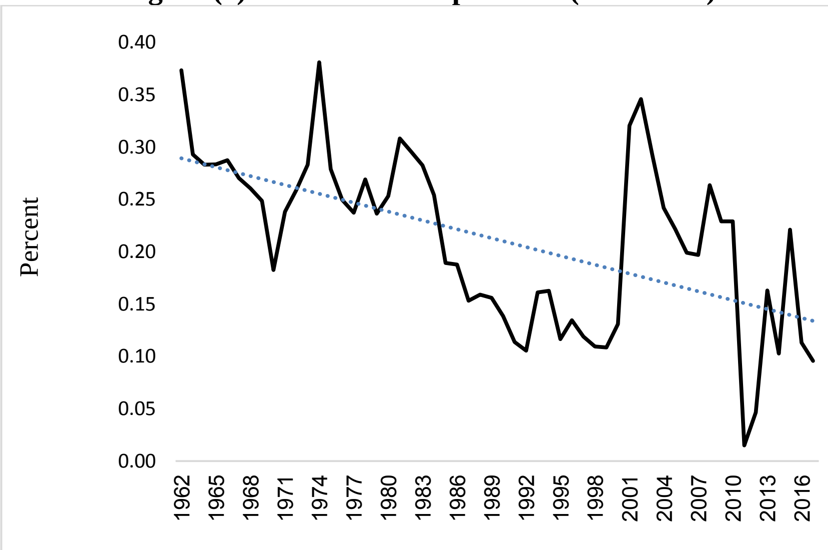
Source: Author's work.

B. Labour Force:

Throughout the study period, it is noticed that labour grew at a faster rate compared to that of population, while capital growth rate was declining except in some years while the growth rate of capital was declining. This led in turn to a decrease in per-worker–capital ratio and greater capital productivity relatively. The increase in the labour force had been created by the expansion of employment in the public services at the expense of productivity as noticed by world bank (World Bank, 2006). Abuhadra & Ajaali (2014) also asserted this result which leads to declining in labour productivity.

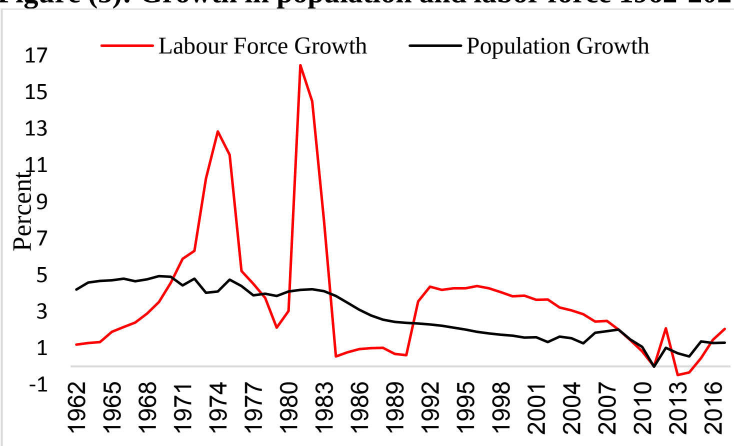
The reason of the declining trend can be understood on the investment/output ratio which also showed a decreasing over the period; however, it reflected mainly the oil prices booms as seen in figure (4) die to decline in percentage dedicated to investment.

Figure (4): Investment output ratio (1962-2020)



Source: Author's work depends on Libyan and population census.

Figure (5): Growth in population and labor force 1962-2020



Source: Author's work depends on Libyan and population census.

As a result of prosperity, labour force grew at 3.54% on average, which is higher than that for population, and remained higher for most years. This led to increasing employment over the years later, when oil prices collapsed. This may explain the diminishing trend in labour productivity over time, which in turn denotes the low significance of labour factor in economic activities as stated by the (World Bank, 2006).

C. Human Capital:

Human capital is proposed as a competitor to technological progress to account for Solow residual. By the late 1950s, human capital had been paid more attention however, it dated back to Adam Smith who pointed out that *"The improved dexterity of a workman may be considered in the same light as a machine or instruments*" (Smith, 1910, p.49). Human capital is considered a driving factor for economic growth alongside physical capital (Breton, 2014; Fadi, 2014; Pelinescu, 2015; Boztosun et al., 2016) However; there is significant debate as to whether this factor can be sustained and passed to new generations.

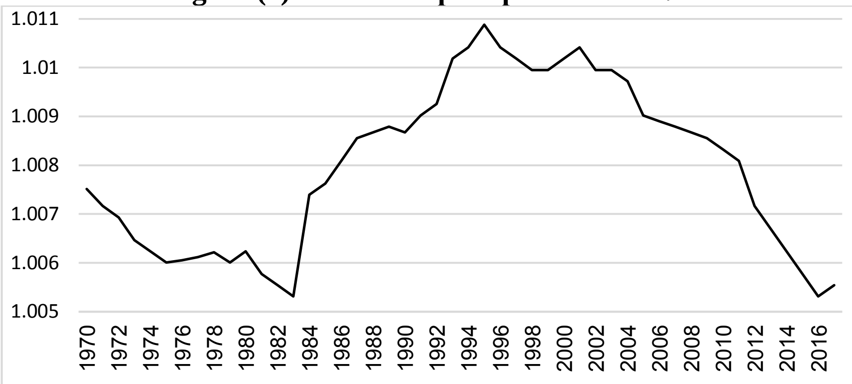
Moreover, Stiglitz et al. (2009) argued that measuring the role of human capital in well-being levels is controversial because human capital itself remains elusive, physical indicators of human capital are applied to provide a comparison.[2] However, these indicators have critical limits (Stiglitz et al., 2010). Regards our case, Libya, there were substantial restrictions hindering efforts to improve human capital Before the 1960s. According to WB data, education system was very primitive, and Libyans had no formal schooling for a long time. There was however, widespread semi-school education, which private institutions provided, and most of these were joined to mosques, where illiteracy was taught were higher than for most underdeveloped countries, with little genuine human capital stock during that time (Higgins, 1968). After independence, the government provided education to every child, but financial constraints were a difficult barrier. The big push in education came after oil discovery in the 1960s. Mandatory nine-year schooling was sought and free education also provided along with most developing countries (Hamdy, 2007, p.2). Therefore, enrolment exceeded 100 per cent in many years[3], and the illiteracy rate had fallen sharply consequently, despite the significantly growing birth rate (Campante & Chor, 2012). In the seventies, education policy turned to a technical and vocational, along with high investment in industrial institutions, which promoted demand for technicians. These policies influenced the accumulation of human capital stock. However, the highest marginal return for education is at eight years of schooling, this indicates the importance of primary

2- Human capital indicators such as the average years of schooling for the population of working age.

3- Because many poor family's children were out of schools before 1960s, late-ages enrolment pushed the rate of enrolment above 100 percent when enrolment became compulsory.

education in developing countries like Libya rather than higher education (Arabsheibani & Manfor, 2001).

Figure (6): Human capital per worker H/L

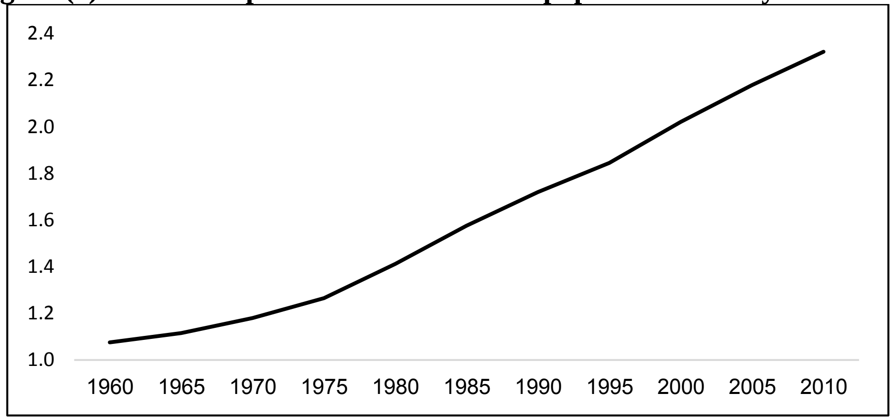


Source: Author's work. Human capital per-worker is calculated as follows: Human capital index HCI is multiplied by the population in the corresponded year then divided by the number of the labour force in the same year.HCI is obtained from WB and available on: https://datacatalog.worldbank.org/dataset/human-capital-index

Consequently, human capital per-worker showed an increase in the 1980s, followed by a decrease after 1995, as seen in graph (6). This can be explained by the long-run effect of insufficient investment in education and healthcare, as stated by Humphreys et al. (2007), Human capital also witnessed many fluctuations which were associated with oil shocks and showed a reversed trend with the physical one. Despite the rarity of data and studies on human capital in Libya, Barro and Lee provided a human capital stock (HCS)[4] for a series of five-year periods from 1950 up to 2010. It provides a reliable indicator of the progress made in such factors after oil was discovered, as shown in Figure (7).

4- To know more how HCS is measured, one can see Ann Lisbet Brathaug 'Guide on Measuring Human Capital' Summary from UNECE Task Force Guide on Measuring Human Capital, World statistics Congress Morocco July 16-21, 2017, available on-line at: http://old.iariw.org/marrakesh/BrathaugPDF.pdf

Figure (7): Human capital Stock for the total population in Libya 1960-2010



Source: Lee and Lee, Long-run education dataset, http://www.barrolee.com/Lee Lee_LRdata_dn.htm, accessed: May 6, 2020, 19:45.

Overall, Collins and Bosworth (1996), Rauch and Meier (2000), Pritchett (1999), and Campante and Chor (2012) all argued that the role of human capital in developing countries, and in oil-based ones in particular, is still reasonably doubtful.

To conclude, it can be stated that human capital remains under reasonable doubt in terms of its role as an input factor in economic growth, and as a proxy for input variable in developing countries due to the lack of data. In this context, average years of schooling has been included in the production function as a human capital. However, Barro and Lee (2013) prefer the human capital adjusted[5] to return to schooling over average years of schooling.

D. Total Factor Productivity TFP:

Studies on Total factor Productivity within developing countries and oil-based ones, in particular, are still rare, mainly due to lack of data (Barro & Lee, 2013). Moreover, the outcomes mostly contradicted each other. However, Pritchett (1999) and Senhadji (2000) agreed that TFP in MENA countries witnessed a decreasing trend after the 1970s.

For Libya, the available studies on this issue are very rare and outcomes are as follows. Eltaief and Ahmad (2011) argued that growth of TFP was mostly due to technical change rather than to labour productivity. Arab Planning Institute supported these outcomes by stating that productivity had grown by -

5- The adjusted-human capital is defined as the gross enrolment ratio minus the proportion of repeaters. If the gross enrolment ratio is not available, the net enrolment ratio is used as a proxy for the adjusted enrolment ratio

2.3% between 1995 and 2016. Libya recorded the second-lowest rate for the Arab countries in this period (API, 2018).

Table (1): TFP growth in Libya 1990-2016

Period	Growth Rate	Source
1962-2006	-8.3	(Masoud & Alkaa'ida, 2014)
1991-2001	-0.2	(IMF, 2006)
1990-2012	10	(Eltaief & Ahmad, 2011)
1995-2016	-2.3	(API, 2018)
2000-2009	0.17	(Fargani, 2013)
1991-2000	-8.1	(Ali, 2016)
2001-2006	-6.2	(Ali, 2016)

Source: Arab Institute for Planning, Arab Development Report (2018), (Ali 2016)

3. Methodology and Data:

This study will apply both the Solow-Exogenous growth model and the endogenous growth model proposed by Mankiew, Romer and Weil given the availability of national accounts data and because this approach has been widely used in the literature as the Solow model has enough substance to apply, as Acemoglu (2008) noted. In addition, it will apply the assumption of Hicks-neutral technical change with constant returns to scale (CRS). The Cobb-Doglas production function will also be employed. It is noteworthy that many formulas can be applied to estimate the trend of growth for any economy; however, it is common to use the CD formula to estimate such economies (Barro & Sala-i-Martin, 1995). Starting with the assumptions of the Solow model with CD production function, there are two inputs, capital and labour. Population and technology growth rates are exogenously determined. The production function is under CRS assumption.

For this purpose, the analysis will follow regression approach and utilise GDP as the principal growth indicator applying (MRW) formula as suggested by Mankiw, Romer, and Weil's (1992) as follows:

$$ln\left(\frac{Y_t}{L_t}\right) = ln\,A_0 + \lambda_t + \frac{\alpha+\beta}{1-\alpha-\beta}\ln(s_t) - \frac{\alpha}{1-\alpha}\,ln\,(\eta+\delta+\lambda) + \frac{\beta}{1-\alpha-\beta}\ln(h_t)$$

Where $\frac{Y_t}{L_t}$ is the per-worker output at period t, s_t denotes savings to output ratio: however, it is often considered as a proxy of physical capital, and h_t denotes accumulation of human capital.

Due to its relevancy and having simplified convenient assumptions for underdeveloped economies (Agell et al., 1997; Acemoglu, 2008). The reason behind this choice is that this approach allowed us to investigate the role of human capital precisely since this input factor is one of two candidates for explaining economic growth. Also, the suggested MRW tried to address the criticisms which face the original Solow model and touches the influence of savings magnitude as well as population growth on output (Picketty, 2014). These aspects suit this study as the savings in oil-based economies considered one of the crucial variables for economic growth in addition to the relatively high population growth in Libya. It is widely used among literature and empirical studies (Bernanki, 2001; Ding & Knight 2009; Breton, 2013). The authors of MRW model argued that an augmented Solow model, which includes the accumulation of human capital as well as physical capital, offers an excellent tool for understanding and explaining cross-country variations in the standard of living. For this purpose, they suggested an extended model for convergence in these standards across countries. This model is chosen as its empirical results showed that the model provides an effective tool to analyse the process of growth. This method exposes more important for both physical and human capital accumulation, which consistent with the aim of this study. As argued by Breton (2014). The structure of MRW model is consistent with microeconomic evidence for the effects of schooling on economic growth. The assumption of the role of human capital is more compatible with historic cross-country evidence on the economic growth rather than many other similar models like the basic Solow model and Hall & Jones models. Moreover, Breton (2013) claimed that the validity of the augmented Solow model is supported by the importance of human capital which he arrived at. This model focus on the role of institutions in society in the growth process through the impact of physical and human capital accumulation (Eicher et al., 2006). Also, it is able to explain most variation empirically in per capita income on cross-section level (Gasiorek et al., 1992), and both cross-section and individual levels (Ding & Knight, 2009). The main criticism of MRW model is that their assumption on the common exogenous rate of technological growth (Klenow & Rodrıguez-Clare, 1997; Easterly & Levine, 2001; Gundlach, 2007; McQuinn & Whelan, 2007). This assumption is not applied in our case as it is not cross-country type.

Besides, the assumption of Hicks-neutral technical change has been employed to assess sources of growth and determining factors with constant returns to scale (CRS). This assumption is postulated because there is no reason to avoid this assumption for an underdeveloped economy like Libya following (Acikgoz & Mert, 2015; Jones & Scrimgeour, 2005; Acikgoz & Mert, 2014; Narayan & Narayan, 2005).

4. Micro Foundation:

The suggested technique of MRW tries to address criticisms of Solow idea that the sources of growth (i.e. technical progress and population growth) are exogenous. MRW simply augmented the Solow model by including human capital as well as physical capital as a separate input factor. As a result, the model became more endogenous model of growth. Their findings revealed that adding this variable improved predictions and enhanced the magnitude of physical capital, giving the model robustness for the relationship already suggested (Mankiw et al., 1990). In addition, Cobb-Douglas production function will be employed to estimate the contribution of each factor to the growth since this formula suits such economies[6] as Libya's (Rose, 1977). Before estimation, there are two issues worthy to be declared:

First: there are many proxies for physical and human capital, as Mankiw et al. (1992) admit in their empirical study. They suggest two ways to deal with this variable: one is to estimate a reduced form of Equation (23 in Appendix), in which the rate of human capital accumulation represents the variable. The other approach is to estimate Equation (23) directly, where h_t is represented by the stock of human capital.

The second issue with human capital is that of which data will be better for estimation; this is related to the availability of data and its quality.

MRW constructed their data for this variable, which places more conditions in this context. Due to the limitations of data on human capital for most underdeveloped countries, four indicators have been considered and tested as proxies for human capital. They are; the human capital index (HCI), average years of schooling, the literacy rate for people over 15, and expected lifespan.

5. Variables, Data Sources and Pre-Estimation Tests.

A. Variables:

Before estimation, it is necessary to introduce the variables to be used henceforth. As follows:

1. Aggregate output per-worker in real values measured in local currency and U.S. Dollars. This is due to the nature of oil-based economies, GDP has profoundly affected by the oil sector, which accounts for 60-75% of aggregate output and 90% of government revenues on average. These revenues are measured through the exchange rate, which changed remarkably between 1999 and 2002[7]. To avoid the effect of exchange rate on GDP, we used GDP in U.S. dollars in real values. Since other associated

6- There are many formulas in which we can apply to estimate the growth for any economy, however, it is common to use the Cobb-Douglas for estimating under developing economies see for example: Barro, R. & Martin, X. Economic Growth, 1995, p.150

7- At the end of 1990s the value of Libyan Dinar was affected by the black-market and it lost more than 30% of its nominal value. To tackle this problem, the government devaluated the Libyan currency (Dinar) by 50% in 1999, then by 50% in 2002. See Merza, 2012 pp.197-200 variables are presented in ratios, it is assumed that this action will not affect the results.

2. Non-oil output per-worker (Production in all sectors out of the oil), in local currency and U.S. Dollars.
3. Non-oil output per-capita (Production in all sectors out of the oil).
4. Investment/output ratio obtained by dividing annual investments by aggregate output.
5. The saving to output ratio, calculated by dividing annual savings by output of the correspondent year.
6. Capital stock per-worker, (perpetual capital stock) calculated as follows: adding annual investment to the previous capital stock then subtracting the annual depreciation rate.
7. As per the term $\boldsymbol{ln(n+g+\delta)}$, Mankiw, Romer and Weil assumed that technological progress $\boldsymbol{g}$ is equal to **0.02**, and $\boldsymbol{\delta}$ is equal to **0.03**[8] as an average for all countries. Both are added to population growth for each country (Mankiw et al., 1992). In this study, this variable is represented by $\boldsymbol{ln(n+g+\delta)}$, which is the sum of calculated depreciation rate, annual population growth and technological progress as in the MRW model.

Depreciation rate was calculated depending on the annual capital stock throughout the studied period and equals 0.039. Previously, Zarmouh (2010) estimated it at 0.059 for 1962-1995. Therefore, we will employ the newly calculated rate of 0.039 because it is moderate and includes a more extended period.

B. Data sources:

Data is collected for all variables covering the period 1962-2020 from local sources named: central bank of Libya, the economic research centre at Garyounis University, Statistics and census authority.

All variables will be in logs of real values in Libyan currency in 2010 prices unless stated. They are illustrated in the table below.

For estimation, fully modified ordinary least squares FMOLS will be employed. This method deals with subsections in time-series problems such as serial correlation and endogeneity Philips and Hansen (1990).

8- They assumed that technological change and depreciation rate are similar across the countries and there is no reason to assume these two variables are different across different countries.

Table (2): Variables employed for estimation

Variables	Definition
Dependent variable	
GDP per worker in local currency	ln (GDP/W)
GDP per worker in U.S. Dollars	ln (GDP$/W)
Non-oil GDP per worker in local currency	ln (NGDP/W)
GDP per worker in U.S. Dollars	ln (NGDP$/W)
Non-oil GDP per-capita in local currency	ln (NGDP/Pop)
Independent Variables	
Capital stock per worker	ln (CapStk/W)
Saving output ratio	ln(S/Y)
Investment to output ratio	Ln(I/Y)
Population growth+ depreciation rate+ technology*	$\ln(n+g+\delta)$

** Mankiw, Romer and Weil assumed that A grows at 0.02, and equal for all untries.*

Within each estimated formula, there is a proxy of output-aggregate GDP per worker in both local currency or U.S. Dollars, or non-oil GDP per worker also in both local currency or U.S. Dollars as a dependent variable. While independent variables are as follows: capital *(k)* is represented by one of three proxies: savings to GDP ratio, investment /output ratio and capital stock per worker.[9] In addition to $ln(n+g+\delta)$.

C. Pre-estimation Tests:

Before estimating, ADF and P.P tests was conducted for stationarity.

Table (3): Variables stationarity test

Test Variables	A.D. F		P.P.		Result
	I (0)	I (1)	I (0)	I (1)	
ln(GDP/W)	-1.09	-6.62***	-1.63	-6.65***	I (1)
ln(GDP$/W)	0.37	-6.80***	0.31	-6.80***	I (1)
ln(NGDP/W)	-1.41	-7.02***	-1.57	-7.02***	I (1)
ln(NGDP$/W)	0.37	-6.70***	0.31	-6.8***	I (1)
ln(NGDP/Pop)	-2.47*	-7.54***	-2.26*	-7.53***	I (1)
ln(Capstk/W)	-2.54	-4.46***	-2.68*	-4.44***	I (1)
ln(S/Y)	-3.68***	-9.78***	-3.96***	-11.17***	I (0)
ln(I/Y)	-3.25**	-9.13***	-2.99**	-12.58***	I (1)
$\ln(\mathbf{n}+\mathbf{g}+\boldsymbol{\delta})$	-0.57	-3.12*	0.34	-7.85***	I (1)

Lag length for A.D.F. test is Shwartz info criterion while spectral estimation method for P.P. test is Barlett kernel

*N.B. * Significant at 90%, ** Significant at 95% , *** Significant at 99%*

9- In the MRW empirical study, only two proxies have been applied; savings to GDP ratio ($^S/_y$), and capital stock per worker ($Stck/W$).

Table (3) summarises the results. It shows that all variables of interest are stationary at the first difference at 99% of confidence according to both tests except the variable *ln* $(\boldsymbol{n}+\boldsymbol{g}+\boldsymbol{\delta})$, which is stationary at 95% of confidence according to ADF.

Accordingly, and following (Acikgoz & Mert, 2015) and (Narayan & Narayan, 2005).

FMOLS technique is applicable to estimate the desired function to capture the parameters of input factors as suggested above.

D. Estimation and Results:

The EViews16 application is used to estimate parameters. Estimation is achieved in two steps; Basic Solow model, then augmented MRW model. In both steps, Aggregate and non-oil output both are employed as dependent variables exchangeable.

First: Basic Solow Model: Basic Solow model, as demonstrated in Equation (19), will be applied.

Aggregate GDP:

Regression showed that saving to output ratio as a proxy for capital input gave results as predicted by the model. As shown in the table below.

Table (4): Basic Solow model estimates for Libyan economy 1962-2020

Dependent variable	***Ln (GDP/W)***
Constant	11.65*** (17.4)
ln(s/y)	0.63*** (3.41)
$\ln(n+g+\delta)$	-0.50*** (-2.60)
Implied α	0.39
N	59
Adj R-sq	0.47
Jarque-Bera normality test (P-value)	0.34 (0.84)
Estimation method	FMOLS

*Note: T-statistics in parentheses *** p<0.01, ** p<0.05, * p<0.1 Implied α for model 1 is calculated following MRW:* $1/(1-\alpha)=-0.63$ *then, α = 0.39*

Coefficients on savings and $ln(n+g+\delta)$ are almost equal in magnitude and have opposite signs as predicted. Statically, R^2 indicates that independent variables can explain up to 50 per cent of changes in per-worker income.

For robustness, and to exclude any effects of implemented policies, the prolonged period is divided into sub-periods according to these policies as listed in chapter three as follows:

1. 1962 to 1980, the period of the increasing dominance of oil on economic activities and liberal policies were taken place.
2. 1981 to 1990, this period was characterised by government control on the economic level and shape fall in oil revenues.
3. 1991 to 2010 witnessed a return to open-door policies and expansion in government spending promoted by high oil prices.

Some forms are estimated and demonstrated in the table (5).

Table (5): Basic Solow Model Estimates for sub-periods

Dependent Variable	Form 1 Ln(GDP/W) 1962-1980	Form 2 Ln(GDP$/W) 1962-1980
Constant	8.42*** *(16.62)*	3.07*** *(4.34)*
$\ln(S/Y)$	0.71** *(2.02)*	
$\ln(\text{Capstk}/W)$		0.90*** *(9.21)*
$\ln(n+g+\delta)$	-1.06*** *(-5.03)*	-0.64*** *(-3.87)*
Implied α	0.51	0.39
N	18	18
Adj R-sq.	0.70	0.95
Jarque-Bera normality test (P-value)	1.52 (0.47)	1.86 (0.39)

*Note: t-statistics in parentheses *** p<0.01, ** p<0.05, * p<0.1*

Results show that the explanatory power of independent variables has increased to 0.73 Also, naturalising the exchange effect by using U.S. Dollars to measure the variables of interest brings the capital stock to be an explanatory variable instead of saving ratio. Determination coefficient and significance of parameters both have enhanced as well. Moreover, explanatory variables still have opposite signs as predicted; however, they are not equal in magnitude as predicted. However, the model becomes unable to explain the changes in the output per-worker after 1980.

Non-oil GDP:

Mankiw, Romer and Wiel excluded oil-based economies from their empirical study (Mankiw et al., 1990). Therefore, this paper addresses this issue specifically.

For this purpose, the non-oil output is employed instead of aggregate GDP. Only per-capita income as a dependent variable gave reliable results for a prolonged period. Then excluding the last seven observations (e.g. instability period 2011-2020) showed that $ln\, {}^{I}/_{y}$ becomes significant over $ln(s/y)$ in explaining changes in the non-oil output. Furthermore, excluding the effect of the exchange rate enhanced the explanatory power of the independent variables R^2 remarkably to 0.68. This is expected for the non-oil output and shows how strong is the impact of oil and exchange rate on the non-oil output. It can be noticed that explanatory power R^2 is higher for the non-oil sector.

Table (6): Basic Solow Model Estimates for Non-oil GDP 1962-2020

Dependent / Variable	Form1 Ln(NGDP/Pop) 1962-2020	Form2 Ln(NGDP/W) 1962-2010	Form3 Ln(NGDP$/W) 1962-2010
Constant	6.01*** *(9.9)*	9.1*** *(21.65)*	-2.67 *(-0.87)*
ln(S/Y)	0.43*** *(4.54)*	-	-
ln(I/Y)	-	0.70*** *(4.16)*	-
Ln (CapStk/W)	-	-	1.54*** *(4.77)*
Ln (n+g+δ)	-0.60*** *(-2.59)*	-0.48*** *(-2.48)*	-0.94*** *(-5.41)*
Implied α	*0.37*	*0.32*	*0.48*
N	*59*	*48*	*48*
Adj R-sq.	*0.36*	*0.34*	*0.68*
Jarque-Bera normality test (P-value)	*2.02 (0.36)*	*0.21 (0.90)*	*0.02 (0.99)*
Estimation Method	*FMOLS*	*FMOLS*	*FMOLS*

*Note: t-statistics in parentheses *** p<0.01, ** p<0.5, * p<0.1*
** Implied α for model 1 is calculated following MRW:* $1/(1-\propto) = 0.60$ *then, α = 0.37*

Second: Augmented Solow Model:

Considering the analysis above, the extended Solow model, Equation (23), has been applied for the same period, and a similar process has been followed. No reliable results can be obtained either due to statistical economic or theoretical criterion. Some of the estimated forms gave positive signs for the term $ln(n + g + \delta)$ and human capital parameters, which contradicts assumptions and the theory. In some cases, the parameters could not pass the necessary tests. Some estimated functions are demonstrated in the Appendices.

6. Discussion:

Compared with MRW results, three aspects support the basic Solow model in explaining changes in per-worker GDP. **First,** coefficients on $ln(n + g + \delta)$ and savings ratio are almost equal in magnitude and have the predicted opposite signs. **Second,** the Saving ratio in the Libyan economy can be considered to explain the changes in per-capita output for both short and long-run periods, while investment to output cannot. For instance, this seems to contradict the MRW outcomes. However, a plausible explanation can be found in the nature of oil-based economies. Savings are strongly related to oil revenues. In the main time, they also related to the level of the output because the oil-extracting is dominating the output in these countries. Therefore, the changes in savings could explain the variations in output while investments cannot. **Third,** when U.S. Dollars used to measure output per-worker, capital stock to output ratio $(Capstk/W)$ becomes statistically significant. This means that the capital stock can be considered as an explanatory variable for output only after neutralising the effect of the exchange rate, which supports the assumption of MRW about the difficulty of suggesting a specific model to explain the changes in output in oil-rich countries.

On the other hand, due to the abundance of oil wealth, capital share in income in the Libyan economy according to the national accounts is high and higher than that in many countries. It averaged 0.68 over the studied period. Therefore, the elasticity of income with respect to capital is expected to be about two-thirds or higher. Implied [$\propto$] for the prolonged period of (1962-2020) is much lower than expected. It recorded about one third for both the aggregate and non-oil output (037 and 0.38). This result contradicts with predictions and even contradicts MRW, who arrived at higher values for $\propto$ than predicted (Mankiw et al., 1992). This contradiction can be explained by the capital unproductivity in the Libyan economy, where the share of capital is much higher than its contribution due to the oil-sourced income.

Return to Table (6); the model becomes unable to explain changes in income after 1980. There were remarkable changes in the Libyan economy started by the end of 1970s[10] and became effective by 1980s. Therefore, such results are not surprising because the changes in income during 1980-2020 were influenced strongly by two significant events than that suggested by the model: the global oil market and unstable government policies.

10- In 1978, Libyan authorities started socialist policies and nationalised the most popular economic activities i.e. retails and distribution, then followed by other ones. By 1980, most economic activities became governmental operated.

For the non-oil GDP, investment and saving ratios are both statistically significant, which could be considered as proof that both variables would tend to converge in the non-oil sector.

Also, while the elasticity of the aggregate output with respect to capital $\propto$ recorded about 0.5 for the period (1962-1980), it decreased sharply when the effect of the exchange rate is naturalised. Moreover, this elasticity is slightly higher for the aggregate GDP than that for the non-oil GDP. This difference can be understood in two ways: relatively low labour productivity in the non-oil sector, higher productivity of labour in the oil sector, due to two reasons: high technology in the oil sector which helps in enhancing labour productivity; and the large quantity of capital in the oil industry,[11] which leads to higher labour productivity as stated by the law of increasing returns. The sum of the coefficients of input factors estimated using time-series are not equal to unity because the input factors are not paid the marginal productivity of each, this is the essential condition of the Solow model.

When the extended model was applied, the results were not reliable. This is consistent with several studies for many oil-based countries, as stated by Zarmouh (2010) for Libya and Algeria by Ahmed (2013). The plausible explanation is because the increase in human capital stock, in the long run, was higher than technological progress, and human capital will show diminishing returns (Barro, 1996). In this regard, Benhabib and Spiegel (1994) stated that human capital features are insignificantly in explaining per capita growth, and suggested an alternative approach for human capital in influencing the growth of TFP, they obtained positive results.

The World Bank has warned that natural resources may hinder the process of human capital accumulation as a base of long-run economic growth (Cockx & Francken, 2015). The question to be addressed here is why the MRW model works well in the basic form and does not in the extended form. It is considerable to note that Mankiw, Romer, and Weil have excluded oil-based countries from their empirical study, noting that standard growth models may not be reliable for explaining the economic process for extraction activities. The fundamentals of economic growth require a relationship between input factors and output, which does not exist in oil extraction operations, or at least does not work properly.

Another explanation was suggested by (Basu & Bhattarai, 2012), they argued that the role of human capital is still unclear or insignificant in developing countries, either due to the quality of data, or the low impact of education on economic growth in these countries. This feature does not appear

11- It is well known that the oil industry is one of the most capital-intensive industries.

when only physical capital is employed because the MRW model itself is based on this point. Meanwhile, some studies have shown a significant negative impact of oil export dependence on human capital (Blanco & Grier, 2012). Some researches argued that the presence of more siblings in a household -as in most developing countries including oil-based ones- reduces parental investment, which in turn lowers potential utilising the education system in the future (Attanasio et al., 2015).

Moreover, because of the low level of human capital relative to physical capital in oil-based countries and due to progress in human capital because of oil revenues, causality between output and human capital is still unclear. Oil revenues help remarkably in building the education and health system, causing human capital development. Therefore, the causality relation may exist oppositely to that in developed countries.

Alternatively, average years of schooling is widely considered and employed as a proxy for human capital. For our case, this variable has improved steadily over the period. Libya has been noted, along with most MENA countries, as among the fastest 20 countries in an expansion of schooling from 1960-2000 (Pritchett, 1999; Barro & Lee, 2011), placing sixth among 145 countries in the increase in average years of schooling from 1.5 years in 1970 to 8.2 years in 2010 (Arab Monetry Fund, 2015, p.251). However, this proxy has been criticised because it overstates growth in human capital in underdeveloped countries with relatively low initial levels of education. Some studies have even found that it is difficult to detect a significant impact of changes in years of schooling on economic growth Collins & Bosworth (1996) and even controversial to do so (Rauch & Meier, 2000; Pritchett, 1999). In addition, according to Campante and Chor (2012), there was no apparent correlation between change in schooling years and the unemployment rate in most oil-rich countries.

7. Conclusion:

Statistically, it can be claimed that basic Solow model is more relevant tool for explaining changes in income per-worker in an oil-based economy like Libya's. This model is more appropriate for the short run than for the long run. Moreover, the share of capital is much lower than predicted. On the contrary, we found no evidence for the MRW formula (augmented Solow model with human capital) in explaining the output changes. To understand these outcomes, we ought to remember the note of Mankiw, Romer, and Weil that one should not expect a standard growth model to explain the economic process for the extraction process. The fundamentals of economic growth require a clear relationship between input factors and the output, this relationship does not exist in the oil extraction procedure or at least does not work properly. Moreover, the causality between output and human capital is still unclear due to the low level of human capital compared with physical one because the oil revenues support the accumulation of physical capital in the expense of human one. In fact, the oil revenues helped remarkably in education and health system building and ended up in human capital developing, but this needs more time to work. This is a plausible answer of the question, why the basic Solow model works well and does not in the MRW formula and confirms the basic claim of Mankiw, Romer, and Weil about oil-based economies.

Finally, it is worth to suggest more researches on oil-based economies applying MRW model to confirm these outcomes or reject them. It is also worthy to indicate the unclear role of human capital in under developing countries which needs more investigations.

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Appendex

Derivation of Basic Solow Model:

Starting with assumptions of the Solow model as specified in a typical formula of Cobb-Douglas production function.[12] This formula

- There are two inputs, capital, and labour.
- Population and technology growth is exogenously determined.
- The production function is under a CRS assumption.

$$Y_t = K_t^{\propto}(A_t L_t)^{(\beta)} + e^{ut} \quad ..... \quad (1)$$

Where Y_t stands for annual output represented by real GDP in year *t* as usual.

K and *L* are the input factors, capital and labour, respectively.

($\propto$) and (β) are shares of capital and labour in output *Y* respectively, and both are less than unity $0 < \propto, \beta < 1$. The condition of constant returns to scale is exposed here, so the sum of these is equal to one, $\propto + \beta = 1$.

(A) denotes the level of technology used in the country for the economic process. And e^{ut} is the error term.

According to Mankiw et al. (1990), Solow's growth model considers the growth rates of saving and population as exogenous; therefore, output at any time can be defined by:

$$Y_t = (K_t^{\propto})(A_t L_t)^{1-\propto} \quad ... \quad (2)$$

Labour *(L)* and technology *(A)* are also both assumed to grow exogenously at η and λ respectively, and both are related to time rather than to endogenous changes; thus, they grow as follows:

$$L_t = L_0\, e^{\eta t} \quad , \quad \text{and} \quad A_t = A_0 e^{\lambda t}$$

L_t, A_t are the levels of both labour factor and technology at year *t* respectively, while L_0 and A_0 are the levels of both at the starting period. $e^{\eta t}$and $e^{\lambda t}$ rates ate, which both labour and technology grow over time. The two formulas define the labour supply and the available level of technology at any time.

Assuming competitiveness means that input factors are both paid their marginal productivity, and based on the Euler exhaustion theorem, these payments are equal to output (Dwivedi 2005). Thus, to simplify: $rK + wL = Y$

where r and w are the rates of which capital and labour earn their shares in output.

Dividing by Y, $\frac{rK}{Y} + \frac{wL}{Y} = 1$

Given that $\propto = \frac{rK}{Y}$, $\beta = \frac{wL}{Y}$

12- Derivation of the model can be reviewed in, Robert Solow, (1956), A Contribution to the theory of Economic growth, The Quarterly Journal of Economics, Vol. 70, No. 1 (Feb., 1956) pp.65-94. And in Mankiw n, Romer D, Weil D (1992) A Contribution to the empirics of economic growth, Q. Journal of Economics, 107(2) pp. 407-437. Also it can be found in: Bhattarai, K. (2008). Economic theory and models: Derivations, computations and applications for policy analyses, Serials Publications.

$$\text{then, } \propto = \frac{\propto AK^{\propto-1}L^{\beta}.K}{Y} \text{ and } \beta = \frac{\beta AK^{\beta}L^{\beta-1}.K}{Y}$$

Given the assumption that labour and technology are exogenous (Mankiw et al. 1990), this implies that only capital will be under consideration in this model. Thus, net capital stock is often defined as investment resulting in savings accumulation *(S)* after reduction by the annual rate of capital depreciation (δ), and for equipping new workers at the level of existing workers. As per the neoclassical model, capital plays a substantial role in the process of economic growth. As this is the case, output Y_t depends, along with other factors, on how the corresponding society can accumulate and manage capital (Jones 1998). Savings is the main channel for accumulating capital continuously, and this is just a fraction of output after consumption. Therefore:

$$S_t = sY_t \quad : \qquad 0 < s < 1$$

Here, s_t is savings to GDP ratio at year *t* and must be greater than the sum of the two opposite factors, denoted by depreciation δ and population growth rate *n*.

Therefore: $S = sY_t > (n + \delta + \lambda)K_t$ .... (3)

Growth in capital is equal to: $\dot{k} = \widehat{sy} - (n + \delta + \lambda)k_t$ .... (4)

Equation (4) known as the fundamental equation of growth in the neoclassical model, and $\dot{k}$ is the growth rate of capital with respect to time, equal to dk/dt, while $\widehat{sy}$ is the per-capita savings ratio.

Positive or negative changes in capital must be reflected in per capita capital up to the steady-state level when it becomes zero: $\frac{dk/dt}{k_t} = \frac{\dot{k}}{k_t}$ = zero

Per capita output $\tilde{y} = (\frac{sA}{(n+\delta)})^{\frac{\alpha}{1-\alpha}}$ .... (5)

The necessary market clearance condition is obtained from equating income yielding to income spending as follows:

$$Y_t = C_t + S_t = I_t + C_t \quad \ldots\ldots \quad (6)$$

Where C_t, S_t and I_t are the levels of consumption, savings and investments at period *t;* therefore, the investment must equal savings to obtain equilibrium on the macro-level.

$$I_t = S_t$$

Changes in capital stock can be defined depends on investment and depreciation as follows:

$$dk = I = sY - \delta K \quad \ldots\ldots\ldots\ldots (7)$$

Where dk is the changes in the capital stock periodically and by replacing *Y* by its expression in equation (2) we obtain:

$$dk = I = sAK^{\propto}L^{\beta} - \delta K \quad \ldots\ldots\ldots \quad (8)$$

Given capital at any given period, and adding this to previously accumulated stock gives capital stock at the time *(t)*:

$$K_t = K_{t-1} - \delta K + sAK^{\propto}L^{\beta} \quad \ldots\ldots\ldots (9)$$

$$K_t = (K_{t-1} - \delta K_{t-1}) + I_t \quad \ldots\ldots\ldots\ldots (10)$$

$$K_t = K_{t-1}(1-\delta) + I_t \quad \ldots\ldots\ldots (11)$$

It can also be defined as changes in input factors over time as follows:

$$\frac{dk}{k} = \frac{dK}{K} - \frac{dL}{L} - \frac{dA}{A} \quad \ldots\ldots\ldots\ldots (12)$$

Capital per-worker (*k*) growth can be defined as follows:

Therefore: $$\frac{dk}{k} = \left[\frac{sY}{K} - \frac{\delta K}{K}\right] - \eta - \lambda \quad \ldots\ldots\ldots.. (13)$$

And $$\dot{k} = sy_t - (\eta + \delta + \lambda)\, k_t \quad \ldots\ldots (14)$$

This formula is widely applied for this task and has often been employed chiefly for developing economies (Senhadji 2000; Pritchett 1999). This means that capital stock is affected by population growth (η), as well as by rate of depreciation (δ), and the changes in technology used (λ) and s is the fraction of disposable income dedicated to investment through the savings process.

This defining equation is the core element of the Solow model for growth. Solow considers the fraction of income saved as a policy variable (Bhattarai 2004), as capital per worker-and so capital stock-converges to a steady-state level over time, as follows:

Investment is the unique source for capital formation as in equation (7) even it would be influenced by the terms (η, δ, λ) negatively.

At the steady-state level, there are no changes in the capital (in mathematical language, the F.O.C. equals zero):

$$\frac{\partial k}{\partial t} = s - (\eta + \delta + \lambda) = 0 \quad \ldots. (15)$$

Therefore, the above equation can be written as follows:

$$s = (\eta + \delta + \lambda) \quad \ldots\ldots.... (16)$$

This means that as the savings rate is equal to the sum of population growth, depreciation rate and improvement in technology, the capital stock will not change. Otherwise, the growth path goes in a positive or negative direction depending on whether the RHS of Equation (16) is less or greater than LHS, respectively.

The amount of capital, which fulfils the steady-state condition, k^* is defined as:

$$k^* = \left[\frac{sA}{(\eta+\delta+\lambda)}\right]^{\frac{1}{1-\alpha}} \quad \ldots\ldots (17)$$

For simplicity, output per active worker is preferred over output: therefore, by dividing equation (1) by *AL*, a simple formula is obtained to estimate the model parameters:

$$y = \frac{Y}{AL} = \frac{AK^{\propto} l^{\beta}}{AL} = k^{\propto} l^{\beta-1} = \left[\frac{k}{l}\right]^{\propto} \quad \ldots\ldots (18)$$

Substituting the steady-state level of capital (k^*), Equation (17) in Equation (2) with some arrangements and taking natural logarithms of the terms, steady-state income per capita is found:

$$ln\left(\frac{Y_t}{L_t}\right) = ln\, A_0 + \lambda_t + \frac{\alpha}{1-\alpha} ln(S_t) - \frac{\alpha}{1-\alpha} ln\,(\eta + \delta + \lambda) \quad \ldots. (19)$$

Later, this equation will be presented as the basic model.

Derivation of Augmented Solow Model (MRW):

Mankiw, Romer and Weil (1990) developed their model by involving human capital as an independent variable. They suggested that production function can be decomposed into effective inputs which include capital accumulation *K*, labour efforts *L*, and *H*, human capital (Chambers and Guo 2009).

Their empirical results revealed that adding human capital to the Solow model improved its performance to a predicted direction (Mankiw, Romer et al. 1990). Therefore, the model had been extended to include one more variable *(H)*: human capital[13] as follows:

$$Y_t = A_t K_t^{\propto} H_t^{\gamma} (A_t L_t)^{\beta} \quad \ldots\ldots\ldots\ldots (20)$$

Two types of capital instead of one (physical and human capital) determine the evolution of the economy, and the steady-state levels of both are defined as follows:

$$\dot{k} = sy_t - (\eta + \delta + \lambda)\ k_t \quad \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

$$\dot{h} = hy_t - (\eta + \delta + \lambda)\ h_t \quad \ldots\ldots\ldots\ldots\ldots (22)$$

Where k_t and h_t with dots stands for physical and human capital in the economy at time *t* respectively, while other variables are as before, following a similar process to the previously illustrated, production function with human capital is:

$$ln\left(\frac{Y_t}{L_t}\right) = ln\, A_0 + \lambda_t + \frac{\alpha+\beta}{1-\alpha-\beta}\ln(s_t) - \frac{\alpha}{1-\alpha} ln\,(\eta + \delta + \lambda) + \frac{\beta}{1-\alpha-\beta}\ln(h_t) \quad ..(23)$$

Where $\frac{Y_t}{L_t}$ is the per-worker output at period *t*, s_t denotes savings to output ratio: however, it is often defined as a proxy of physical capital, and h_t denotes accumulation of human capital.

13- Human capital is defined as "*the stock of productive skills, talents, health and expertise of the labour force*" *Human Capital*, Claudia Goldin, (2014), Department of Economics Harvard University and National Bureau of Economic Research. In Oxford English Dictionary, it is defined as "*the skills the labour force possesses… regarded as a resource or asset*."